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DETERMINATION OF THE VELOCITY PROFILE OF A STREAM OF NON-NEWTONIAN FLUID

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We describe a method for determining the velocity profile from measured values of the flow rate in several pipes of various diameters.

The velocity distribution of a fluid over the cross section of a stream can be found rather simply for a known law of flow, i.e., the dependence of the flow velocity on the shear stress. Non-Newtonian fluids differ widely in their rheological behavior. The law of flow is an empirical relation which must be sought separately for each case.

Kutateladze et al. [1] succeeded in separating a rather large subclass of non-Newtonian fluids whose rheological behavior is fairly well described by the linear fluidity law

$$\varphi = \varphi_0(1 + \vartheta\tau). \tag{1}$$

Smol'skii et al. [2] proposed to plot the velocity profile by graphical integration of $dw/dr = f(\tau_R)$, which in turn can be obtained from the Mooney formula by graphical differentiation of the experimental curve of the flow rate as a function of the wall friction stress. The velocity profile for practically any non-Newtonian fluid can be obtained by this method, but it is rather complicated and not very accurate.

The existing methods of measuring flow velocities (photokinetic, electrodiffusion, fluoroscopic, etc.) are complicated both with respect to procedure and with respect to the apparatus used.

We describe a simple method of plotting the velocity profile of the laminar flow of a fluid in a circular pipe. This method is based on the fact that for the same longitudinal pressure gradient the velocity curve in a channel of radius R_1 has the same shape as the portion of the curve for $r \leqslant R_1$ in a larger pipe of radius R_2 (Fig. 1). This conclusion follows from p. 67 of [2].

If the mean flow velocities $\bar{w}_i$ in N pipes of radii $r_i \leqslant R$ are measured for the same longitudinal pressure gradient, the piecewise linear approximation of the curve (Fig. 2) leads to the following relation for the velocity increment between the i-th and (i − 1)-th cross section of the profile:

$$\Delta w_i \approx 3\,\frac{\bar{w}_i r_i^2 - \bar{w}_{i-1} r_{i-1}^2}{r_i^2 + r_i r_{i-1} + r_{i-1}^2}$$

or

$$\Delta w_i \approx 3\,\frac{\bar{w}_i \beta_i^2 - \bar{w}_{i-1}}{\beta_i^2 + \beta_i + 1}, \tag{2}$$

where $\beta_i = r_i/r_{i-1}$. Then the true flow velocity w_n at a distance r_n from the pipe axis is given by the sum

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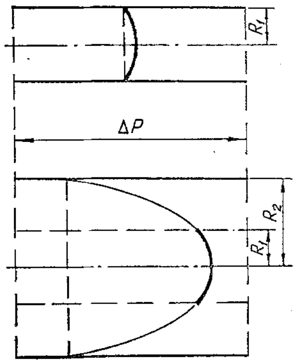

Fig. 1. Velocity profile in pipes of different diameters for the same longitudinal pressure gradient.

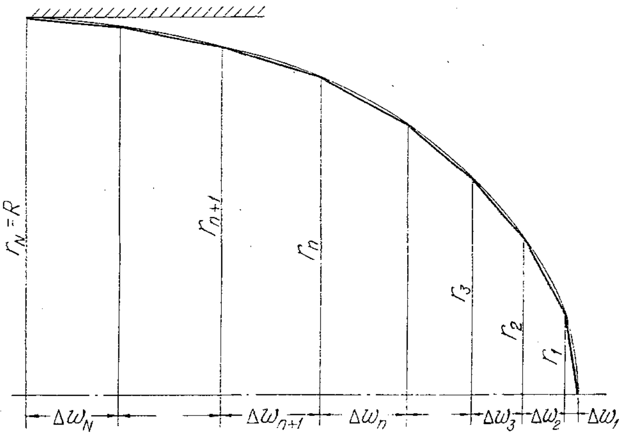

Fig. 2. Piecewise linear approximation of velocity profile (for the derivation of the working formula).

$$w_n = \sum_{i=N}^{n+1} \Delta w_i \,. \tag{3}$$

Thus, in order to plot the velocity profile of a fluid in a pipe of radius R, it is necessary to measure the flow rates Q_i in N pipes of smaller cross section $(r_i \leqslant r_N = R)$ for the same longitudinal pressure gradient, to find the mean flow velocities $\bar{w}_i = Q_i/\pi r_i^2$, and to calculate the Δw_i with Eq. (2). Then the velocity w_n at a distance r_n from the pipe axis is determined from (3).

Using this method the velocity profile was plotted for the flow of a 1.25% aqueous solution of carboxymethylcellulose (CMC). Measurements were performed on ten pipes (N = 10) of different diameters ($r_i \leqslant 9.5$ mm). The same longitudinal pressure gradient (2530 Pa/m) was maintained for all pipes. Table 1 lists the measured and calculated flow velocities.

The velocity profile in a pipe 19 mm in diameter was plotted from the data in the last row of Table 1 (curve 1 of Fig. 3). The figure also shows the velocity profile obtained by the method proposed in [2]. To do this the experimental dependence of the shear stress on the flow velocity gradient was measured with a GDR Reostest-2 rotational coaxial viscometer, and the velocity profile was obtained by graphical integration of the curve $dw/dr = f(r)$. An aqueous solution of CMC obeys the linear fluidity law (1) rather well.

Processing of the experimental results by the method proposed in [1] gives the velocity profile shown by curve 2 of Fig. 3. This method clearly overestimates the velocity. The velocity profile obtained by the method given in [2] deviates from the experimental curve near the flow axis. This results from the fact that the viscometer measurements are inaccurate in the region of small shears. The proposed method gives a velocity profile clearly closer to the actual. The flow rate and the longitudinal pressure gradient can be measured

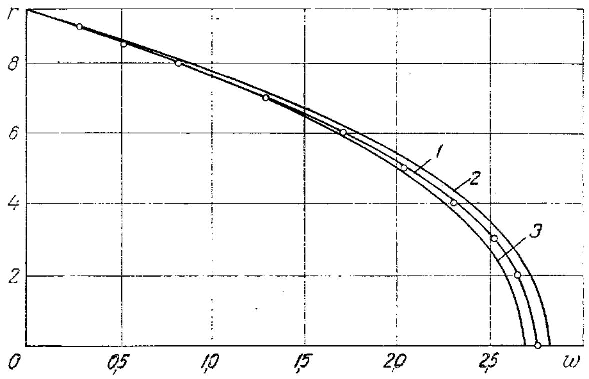

Fig. 3. Velocity profile of flow of 1.25% aqueous solution of CMC w (m/sec) in a 19-mm-diameter pipe: 1) experimental curve; 2) by method of [1]; 3) by method of [2].

TABLE 1. Measured and Calculated Flow Velocities

i	1	2	3	4	5	6	7	8	9	10
r_i, mm	2	3	4	5	6	7	8	8,5	9	9,5
$Q_i \cdot 10^{-6}$, m^3/sec	0,710	3,312	11468	29,525	59,765	115,378	200,650	265,355	318,910	396,905
$\overline{w}_i$, m/sec	0,056	0,117	0,228	0,376	0,528	0,749	0,998	1,169	1,253	1,400
Δw_i, m/sec	(0,112)	0,131	0,210	0,282	0,316	0,418	0,482	0,302	0,228	0,291
w, m/sec	2,654	2,523	2,313	2,031	1,715	1,297	0,815	0,513	0,291	0

very accurately. Therefore, the accuracy with which the experimental curve describes the true velocity profile is determined mainly by the number of pipes and their distribution over diameters. The accuracy of the determination increases with an increase in the number of pipes.

NOTATION

τ, shear stress; φ, fluidity; φ_0, limit of fluidity as $\tau \to 0$; ϑ, measure of structural stability of fluid; R, radius of pipeline; r, radius of cylindrical layer in pipe; w, flow velocity of fluid; τ_R, friction wall stress; Q, flow rate; N, number of pipes.

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